Chapter Ten

Distance and Height

From very ancient times trigonometrical ratios are applied to find the distance and height of any distant object. At present trigonometrcal ratios are of boundless importance because of its increasing usage. The heights of the hills, mountains and trees and the widths of those rivers which cannot be measured in ordinary method are measured the heights and widths with the help of trigonometry. In this condition it is necessary to know the trigonometrical ratios values of acute angle.

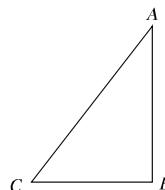
At the end of this chapter, the students will be able to –

- Explain the geoline, vertical plane and angles of elevation and declination
- Solve mathematical problem related to distance and height with the help of trigonometry
- Measure practically different types of distances and heights with the help of trigonometry.

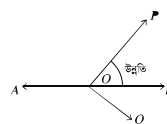
Horizontal line, Vertical line and Vertical plane :

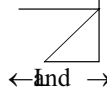
The horizontal line is any straight line on the plane. A straight line parallel to horizon is also called a horizontal line. The vertical line is any line perpendicular to the horizontal plane. It is also called normal line.

A horizontal line and a vertical line intersected at right angles on the plane define a plane. It is known as vertical plane.

In the figure : A tree with height of AB is standing vertically at a distance of CB from a point C on the plane. Here, CB is the horizontal line. BA is the vertical line and the plane ABC is perpendicular to the horizontal plane which is a vertical plane.

Angle of Elevation and Angle of Depression :

Observe the figure, AB is a straight line parallel to the horizon. The points P, O, B lie on the same vertical plane. The point P on the straight AB makes angle $\angle POB$ with the line AB. Here at O, the angle of elevation of P is $\angle POB$.

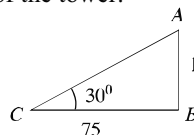
So, the angle at any point above the plane with the straight line parallel to horizon is called the angle of elevation.

Again the point Q, O, B lie on the same vertical plane and point Q lines at lower side of the straight line AB parallel to horizon. Here, the angle of depression at O of Q is $\angle QOB$. So, the angle at any point below the straight line parallel to the plane is called the angle of depression.

Activity :
Draw the figure and show the horizontal line, vertical line, vertical plane, angle of elevation and angle of depression.

N.B. : For solving the problems in this chapter approximately right figure is needed. While drawing the figure, the following techniques are to be applied.

(1) While drawing 30° angle, it is needed base > perpendicular.	
(2) While drawing 45° angle, it is needed base = perpendicular.	
(3) While drawing 60° angle, it is needed base < perpendicular.	

Example 1. The angle of elevation at the top of a tower at a point on the ground is 30° at a distance of 75 metre from the foot. Find the height of the tower.

Solution : Let, the height of the tower is $AB = h$ metre. The angle of elevation at C from the foot of the tower $BC = 75$ metre of A on the ground is $\angle ACB = 30^\circ$

From ΔABC we get, $tan\angle ACB = \dfrac{AB}{BC}$

or, $tan30^\circ = \dfrac{h}{75}$ or, $\dfrac{1}{\sqrt{3}} = \dfrac{h}{75}$ $\left[\because tan30^\circ = \dfrac{1}{\sqrt{3}}\right]$ or, $\sqrt{3}h = 75$ or, $h = \dfrac{75}{\sqrt{3}}$

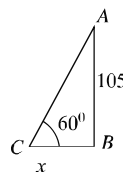
or, $h = \dfrac{75\sqrt{3}}{3}$ [multiplying the numerator and denominator by $\sqrt{3}$] or, $h = 25\sqrt{3}$

$\therefore$ $h = 43.301$ metre (app.).

Required height of the tower is 43.301 metre (app.).

Example 2. The height of a tree is 105 metre. If the angle of elevation of the tree at a point from its foot on the ground is 60°, find the distance of the point on the ground from the foot of the tree.

Solution : Let, the distance of the point on the ground from the foot of tree is $BC = x$ metre. Height of the tree $AB = 105$ metre and at C the angle of elevation of the vertex of tree is $\angle ACB = 60^\circ$

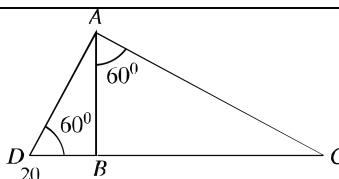
From ΔABC we get,

$tan\angle ACB = \frac{AB}{BC}$ or, $tan60^0 = \frac{105}{x}$ $\left[\because tan60^0 = \sqrt{3}\right]$

or, $\sqrt{3} = \frac{105}{x}$ or, $\sqrt{3}x = 105$ or, $x = \frac{105}{\sqrt{3}}$ or, $x = \frac{105\sqrt{3}}{3}$ or, $x = 35\sqrt{3}$

$\therefore$ $x = 60.622$ (app.)

$\therefore$ The required distance of the point on the ground from the foot of the tree is 60.622 metre (app.).

Activity:
In the picture, AB is a tree. Information from the picture –

1. Find the height of the tree.
2. Find the distance of the point C on the ground from the foot of the tree.

Example 3. A ladder of 18 metres. long touches the roof of a wall and makes an angle of 45^o with the horizon. Find the height of the wall.

Solution : Let, the height of the wall AB is $= h$ metre, length of ladder AC is $= 18$ m. and makes angles with the ground $\angle ACB = 45^\circ$.

From ΔABC we get, $\sin\angle ACB = \frac{AB}{AC}$

or, $sin45^\circ = \frac{h}{18}$

or, $\frac{1}{\sqrt{2}} = \frac{h}{18}$ $\left[\because sin45^0 = \frac{1}{\sqrt{2}}\right]$ or, $\sqrt{2}h = 18$ or, $h = \frac{18}{\sqrt{2}}$

or, $\sqrt{2}h = 18$ or, $h = \frac{18}{\sqrt{2}}$

or, $h = \frac{18\sqrt{2}}{2}$ [multiplying the numerator and denominator by $\sqrt{2}$]

or, $h = 9\sqrt{2}$

$\therefore$ $h = 12.728$ (app.)

Therefore, required height of the wall is 12.728 m. (app.).

Example 4. A tree leaned due to storm. The stick with height of 7 metre from its foot was learned against the tree to make it straight. If the angle of depression at the point of contacting the stick on the ground is 30°, find the length of the stick ?

Solution : Let, the height of the stick from the foot learned against the tree of $AB = 7$ metre and angle of depression is $\angle DBC = 30^\circ$

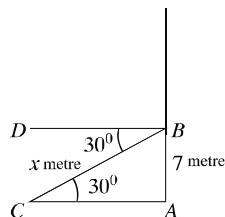
$\therefore \angle ACB = \angle DBC = 30^\circ$ [alternate angle]

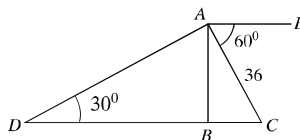
From ΔABC we get,

$sin\angle ACB = \dfrac{AB}{BC}$ or, $sin30^0 = \dfrac{7}{BC}$

or, $\dfrac{1}{2} = \dfrac{7}{BC}$ $\left[\because sin30^0 = \dfrac{1}{2}\right]$

$\therefore BC = 14$

$\therefore$ Required height of the stick is 14 metre.

Activity :

In the figure, if depression angle $\angle CAE = 60^\circ$, elevation angle $\angle ADB = 30^\circ$, $AC = 36$ metre and B, C, D lie on the same straight line, find the lengths of the sides AB, AD and CD.

Example 5. The angle of elevation at a point of the roof of a building is 60° in any point on the ground. Moving back 42 metres from the angle of elevation of the point of the place of the building becomes 45°. Find the height of the building.

Solution : Let, the height of the building is $AB = h$ metres. The angle of elevation at the top $\angle ACB = 60^\circ$. The angle of elevation becomes $\angle ADB = 45^\circ$ moving back from C by $CD = 42$ metres.

Let, $BC = x$ metre

$\therefore BD = BC + CD = (x + 42)$ metre

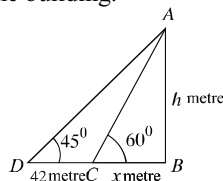
From ΔABC we get,

$tan60^0 = \dfrac{AB}{BC}$ or, $\sqrt{3} = \dfrac{h}{x}$ $\left[\because tan60^0 = \sqrt{3}\right]$

$\therefore x = \dfrac{h}{\sqrt{3}}$............$(i)$

Again, from ΔABD we get, $\tan 45^0 = \dfrac{AB}{BD}$

or, $1 = \dfrac{h}{x+42}$ $\left[\because tan45^0 = 1\right]$ or, $h = x + 42$

or, $h = \dfrac{h}{\sqrt{3}} + 42$; by equation (i)

or, $\sqrt{3}h = h + 42\sqrt{3}$ or, $\sqrt{3}h - h = 42\sqrt{3}$ or, $\left(\sqrt{3}-1\right)h = 42\sqrt{3}$ or, $h = \dfrac{42\sqrt{3}}{\sqrt{3}-1}$

$\therefore h = 99.373$ (app.)

Height of the building is 99.373 metres (app.)

Example 6. A pole is broken such that the broken part makes an angle of 30° with the other and touches the ground at a distance of 10 metres form its foot. Find the lengths of the pole.

Solution : Let, the total height of the pole is $AB = h$ metre. Breaks at the height of $BC = x$ matre without separation and makes an angle with the other, $\angle BCD = 30^\circ$ and touches the ground at a distance $BD = 10$ metres from the foot.

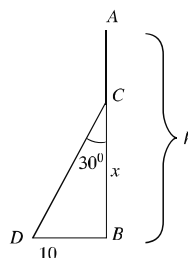
Here, $CD = AC = AB - BC = (h - x)$ metre

From ΔBCD we get,

$tan30^\circ = \dfrac{BD}{BC}$ or, $\dfrac{1}{\sqrt{3}} = \dfrac{10}{x}$ $\therefore$ $x = 10\sqrt{3}$

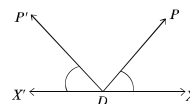
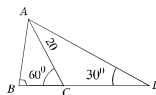
Again, $sin30^\circ = \dfrac{BD}{CD}$ or, $\dfrac{1}{2} = \dfrac{10}{h-x}$

or, $h - x = 20$ or, $h = 20 + x$ or, $h = 20 + 10\sqrt{3}$; [putting the value of x]

$\therefore h = 37.321$ (app.) $\therefore$ Height of the pole is 37.321 metres (app.).

Activity :
A balloon is flying above any point between two mile posts. At the point of the balloon the angle of depression of the two posts are 30° and 60° respectively. Find the height of the balloon.

Exercise 10

1. (a) Find the measurement of $\angle CAD$
   (b) Find the lengths of AB and BC.
   (c) Find the distance between A and D.
2. From a helicopter above a point O between two kilometre posts, the angles of depression of the two points A and B are 60^o and 30^o respectively.
   (a) Draw a figure with short description.
   (b) Find the height of the helicopter from the ground.
   (c) Find the direct distance from the point A of the helicopter.
3. What is the elevation angle of point P from the point O ?
   (a) $\angle QOB$ (b) $\angle POA$ (c) $\angle QOA$ (d) $\angle POB$

4. (i) The horizontal line is any straight line lying on the plane.
   (ii) Vertical line is any line perpendicular to the plane.
   (iii) A horizontal line and a vertical plane define a plane. It is known as vertical plane.

which one is right of the above speech ?

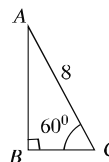
(a) i and ii (b) i and iii (c) ii and iii (d) i, ii and iii

Answer the questions 5 – 6 from the adjacent figure :

5. The length of BC will be –

(a) $\frac{4}{\sqrt{3}}$ m (b) 4m (c) $4\sqrt{2}$ m (d) $4\sqrt{3}$ m

6. The length of AB will be –

(a) $\frac{4}{\sqrt{3}}$ m (b) 4m (c) $4\sqrt{2}$ m (d) $4\sqrt{3}$ m

7. If the angle of the elevation of the top of the miner is 30° at a point on the ground and the height is 26 metres, then find the distance of the plane from the Miner.

8. If the top of a tree is 20 metres distance from the foot on the ground at any point and the angle of elevation of 60°, find the height of the tree.

9. Forming 45° angle with ground A 18 metres long ladder touches the top of the wall, find the height of the wall.

10. If the of depression of a point on the ground 20 metres from the top of the house is 30° then, find the height of the house.

11. The angle of elevation of a tower at any point on the ground is 60°. If moved back 25 metre, the angle of elevation becomes 30°, find the height of the tower.

12. The angle of elevation of a tower 60° moving 60 metres towards a minar. Find the height of the minar.

13. A man standing at a place on the bank of a river observesd that the angle of elevation of a tower exactly opposite to him on the other bank was 60^{o}. Moving 32 metres back he observed that the angle of elevation of the tower was 30^{o}. Find the height of the tower and the width of the river.

14. A pole of 64 metre long breaks into two parts without complete separation and makes an angle 60° with the ground. Find the length of the broken part of the pole.

15. A tree is broken by a storm such that the broken part makes an angle of 30° with the other and touches the ground at a distance of 10 metres from it. Find the length of the whole tree.

16. Standing any where on the bank of a river, a man observed a tree exactly straight to him on the other bank that the angle journey by boat of elevation of he top of the tree of 150 metres length is 30°. The man started for the tree. But he reached at 10 metres away from the tree due to current.

(a) Show the above description by a figure.

(b) Find the width of the river.

(c) Find the distance from the starting point to the destination.